The Vainshtein mechanism in bimetric gravity

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Edvard Mörtsell, Jonas Enander

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Bimetric gravity

$$\mathcal{L} = -\frac{M_g^2}{2}\sqrt{-\det g}R_g - \frac{M_f^2}{2}\sqrt{-\det f}R_f + m^4\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right) + \sqrt{-\det g}\mathcal{L}_{\mathrm{m}}$$

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$$G^g_{\mu\nu} + m^2\sum_{n=0}^{3}(-1)^n\beta_n g_{\mu\lambda}Y^\lambda_{(n)\nu}\left(\sqrt{g^{-1}f}\right) = \frac{1}{M_g^2}T_{\mu\nu}$$

$$G^f_{\mu\nu} + m^2\sum_{n=0}^{3}(-1)^n\beta_{4-n} f_{\mu\lambda}Y^\lambda_{(n)\nu}\left(\sqrt{f^{-1}g}\right) = 0$$

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- Graviton mass

$$m_g^2 = m^2(1 + \frac{1}{c^2})(\beta_1 c + 2\beta_2 c^2 + \beta_3 c^3)$$

where $f_{\mu\nu} = c^2 g_{\mu\nu}$

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- Newtonian gravity in solar system
- Runaway gravitational collapse in early universe?

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- Non-linear structure formation?

Length scales

- Gravitational radius of star (galaxy, cluster...)

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- Vainshtein radius

$$r_V^3 \sim M \lambda_g^2$$

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- Including higher order terms

$$\Phi = \Phi_{\mathrm{N}} \left[1 + k_1 \left(1 + \left(\frac{r_V}{r}\right)^3 + \cdots\right)\right]$$

Gravitational field

- Expand in m_g^2

$$\begin{aligned}\Phi &= \Phi_{\mathrm{N}} + k_2 \frac{r^2}{\lambda_g^2} + k_3 \frac{r^4}{\lambda_g^4} + \cdots \\ &= \Phi_{\mathrm{N}} + k_2 \left(\frac{M}{r}\right)\left(\frac{r}{r_V}\right)^3 + k_3 \left(\frac{M}{r}\right)^2 \left(\frac{r}{r_V}\right)^6 + \cdots\end{aligned}$$

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- To lowest order

$$\Phi = \Phi_{\mathrm{N}} \left[1 - k_2 \left(\frac{r}{r_V}\right)^3\right]$$

Gravitational field

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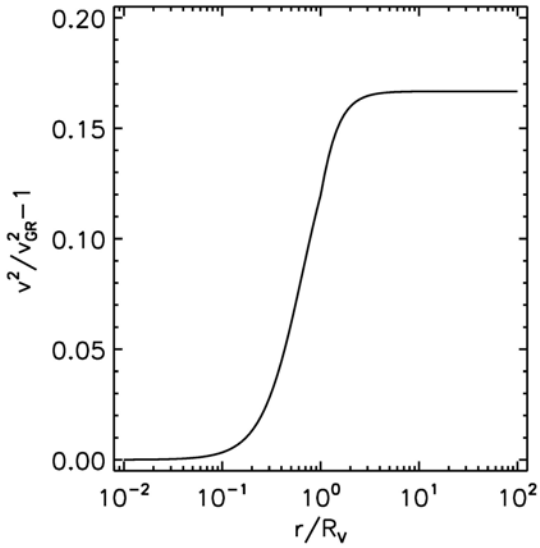
- The Sun has $\Phi(r_V) = 10^{-15}$ and a galaxy $\Phi(r_V) = 10^{-7}$

Galaxy rotation curves



Observational limits

- Deviations from GR in solar system small

$$\left(\frac{1\,\mathrm{AU}}{r_V}\right)^3 \lesssim 10^{-9} \rightarrow \lambda_g \gtrsim 1\,\mathrm{kpc} \approx 2 \cdot 10^{-7}\, r_H$$

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- Galaxy lensing and velocity dispersions

$$\lambda_g \gtrsim 10^{-3}\, r_H \quad \texttt{or} \quad \lambda_g \lesssim 10^{-6}\, r_H$$

Vainshtein density

Average density within r_V of mass $M(r_V)$ is

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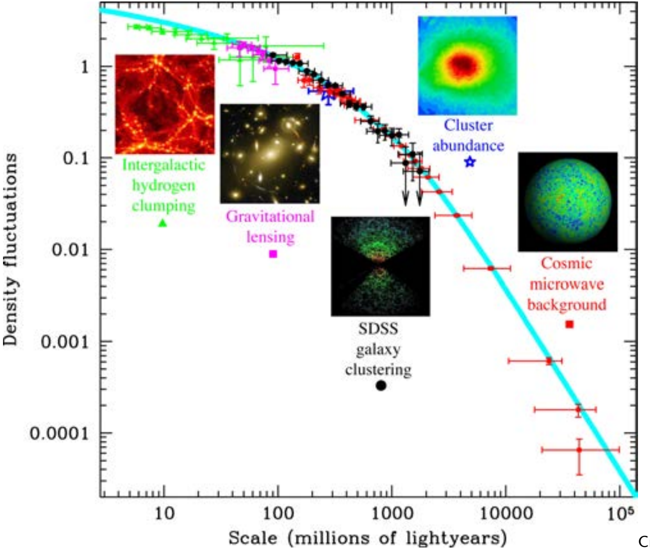
Fluctuation within its Vainshtein radius for

$$\delta \gtrsim \delta_V(a) \equiv \frac{\rho_V}{\bar\rho} \sim \frac{a^3}{\Omega_m}$$

Observational consequences



Credit: Max Tegmark

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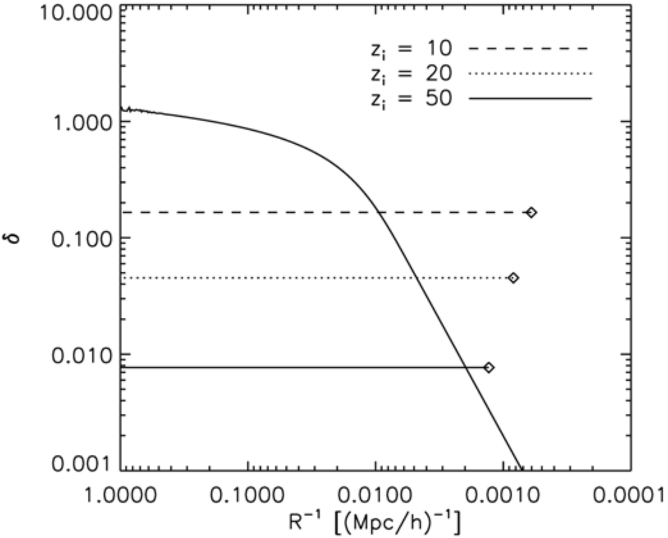

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- Observations at large scales crucial