Institute for Research in Fundamental Sciences

Localized Objects in a 3-Dimensional Chern-Simons Matter Model from a Consistent Truncation of 11-Dimensional Supergravity over $AdS_4 \times S^7/Z_k$

31th IPM Physics Spring Conference (Virtual)

May 15, 2024

M. Naghdi
Department of Physics, Faculty of Basic Sciences, University of Ilam, Ilam, Iran

Abstract

For 11-dimensional (11D) supergravity (SUGRA) over $AdS_4 \times CP^3 \ltimes S^1/Z_k$, we include a new 4-form ansatz, composed mainly of the elements of the internal spcae. Solving the 11D supergravity equations, we obtain a scalar Nonlinear Partial Differential Equation (NPDE) in Euclidean AdS_4 space. The resulting $SU(4) \times U(1)$-singlet (pseudo)scalars arise from probe (anti)M-branes wrapped around the internal space directions in the (Wick-rotated:WR) skew-whiffed (SW) background; and the resulting anti-M2-branes theory breaks all 32 supersymmetries (SUSYs) and parity of the original theory. Taking the backreaction on the external and internal spaces, the resulting bulk equations correspond to exactly marginal and marginally irrelevant boundary operators. Solving the equation, we write a closed solution for the massless ($m^2 = 0$) mode and an approximate solution for a massive mode ($m^2 = 40$) with math methods and especially the Adomian decomposition method (ADM), appropriate for near the boundary analyzes. The solutions have at least the $SO(4)$ symmetry and present instantons responsible for tunneling among almost degenerate vacua of the bulk Higgs-like scalar potential or true-vacuum bubbles growing from the false vacuum in the form of bounce solutions. To realize the bulk singlet (pseudo)scalars and in particular supersymmetry breaking, we excahnge the three fundamental representations for gravitino and as a result, we realize the wished (pseudo)scalars in the spectrum after the branching of $SO(8) \rightarrow SU(4) \times U(1)$. As the same way, using the AdS_4/CFT_3 correspondence rules, by concentrating on the $U(1) \times U(1)$ part of the original quiver gauge group of the 3D boundary Chern-Simons (CS) matter (ABJM) theory, taking just a boundary scalar and a fermion field, introducing dual marginal ($\Delta_+ = 3$) and irrelevant ($\Delta_+ = 8$) boundary operators, and deforming the boundary action with them, we finally find exact solutions with finite actions which are in fact small instantons on a three-sphere with raduis at infinity. In addition, we confirm the bulk state-boundary operator correspondence in the leading order and match elements of the bulk and boundary solutions. Indeed, these solutions are instances of non-SUSY unstable AdS vacua with applications in early universe cosmology, inflationary models and tunnelings (collapsing vacuum bubbles leading to big-crunch singularities).

Scalar Equations in AdS_4 Space from Reduction of 11D SUGRA

Starting from 11D SUGRA in the geometrical background $AdS_4 \times S^7/Z_k$, when the internal space is considered as a S^1/Z_k fiber-bundle on CP^3, we empoly the following anstanz for its 4- form [1]:

$$G_4 = R\, f_1\, G_4^{(0)} + R^4\, df_2 \wedge J \wedge e_7 + R^4\, f_3\, J^2; \tag{1}$$

where e_7 is the seventh vielbein of the internal space, J is the Kähler form on CP^3, $R = 2R_{AdS}$ is the AdS curvature radius, $G_4^{(0)} = d\mathcal{A}_3^{(0)} = N\mathcal{E}_4$ is for the ABJM [2] background with $N = (3/8)R^3$ units of flux quanta on the internal space, $\mathcal{E}_4$ is the bulk unit-volume form and f_i's with $i = 1, 2, 3$ are scalar functions in the external space. Having the anstaz (1), from the Bianchi identity and Euclidean 11D equation, $dG_4 = 0$, $dG_7 - \frac{i}{2} G_4 \wedge G_4 = 0$, we obtain

$$f_1 = \frac{i}{2} R^2 f_3^2 + i\, C_1, \qquad f_3 = f_2 + \frac{C_2}{R} \tag{2}$$

$$\blacksquare_4 f_3 - \frac{4}{R^2}(1 \pm 3C_1) f_3 - \lambda\, f_3^3 = 0, \tag{3}$$

where C_1, C_2 and ... are the real constants, $\lambda = 6$, ■_4 is the Laplacian in EAdS_4 space and the upper and lower sign ($\pm$) behind the sentence containing C_1 shows the WR and SW versions of the background, respectively. Note also that with $C_1 = 1$ (and of course $f_3 = 0$) the ABJM background is realized, and that $\pm(C_2/2) = \pm\sqrt{-\bar{m}^2/\lambda}$ (with $\bar{m}^2 R_{AdS}^2 = (1 \pm 3\, C_1)$) are in fact homogenous vacua and so, the (pseudo)scalar is Higgs-like and the RHS relation in (2) imposes spontaneous symmetry breaking, where f (from now on, $f_3 \equiv f$) acts as fluctuation around the homogeneous vacua.

Taking Backreaction, Resulting Equations and Solutions

Since topological objects such as instantons should not backreact on the background geometry, such solutions are obtained by solving the equations resulting from zeroing the energy-momentum tensors of Einstein's equations with the main bulk equation (3), simultaneously. In fact, from zeroing the external and internal components of the EM tensors of the Einstein's equations, we obtain

$$\blacksquare_4 f + \frac{4}{R^2}(4 \pm 12\, C_1) f + 24\, f^3 = 0, \tag{4}$$

$$\blacksquare_4 f + \frac{4}{R^2}(1 \pm 9\, C_1) f + 18\, f^3 = 0. \tag{5}$$

As a result, from solving the last two equations with the main bulk one (3), we have the equation $\blacksquare_4 f - m^2 f = 0$ (6). In fact, from satisfying the equations (3) and (4) at the same time, that is to include the backreaction of the solution on the external space, we have $m^2 R_{AdS}^2 = 0$, which corresponds to the *exactly marginal* operator in the boundary theory; And in the same way, for solving the equations (4) and (5) with (3), that is taking the backreaction of the whole 11D space, we read the modes of $m^2 R_{AdS}^2 = 1/2$ and $m^2 R_{AdS}^2 = 2/9$ corresponding to the *marginally irrelevant* bounday operators $\Delta_\pm = (3/2) \pm (\sqrt{11}/2)$ and $\Delta_\pm = (3/2) \pm (\sqrt{(89/9)}/2)$. Meanwhile, in upper-half Poincare' coordinates, $ds_{EAdS_4}^2 = \frac{R^2}{4u^2}(du^2 + dx^2 + dy^2 + dz^2)$, an exact solution of (6) reads

$$f(u, \vec{u}) = \bar{C}_{\Delta_+} \left(\frac{u}{u^2 + (\vec{u} - \vec{u}_0)^2}\right)^{\Delta_+}, \quad \bar{C}_{\Delta_+} = \frac{\Gamma(\Delta_+)}{\pi^{3/2}\, \Gamma(\nu)}. \tag{7}$$

Solving the Nonlinear Massive Equation by ADM

We can write the equation (2) by the (conformal) change $f = (u/R_{AdS})\, g$ as follows:

$$\left[\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial u^2} - \frac{(2 + m^2)}{u^2}\right] g(u, r) - \lambda\, g(u, r)^3 = 0, \tag{8}$$

using the spherical coordinates with $r = |\vec{u}|$, $\vec{u} = (x, y, z)$. In fact, for the last equation for f, employing the anstazs as $\xi = u^{1/2} f(r)$ with $f(u, r) = F(\xi)$ (see [2]) and also using the self-similar reduction method (see [3]) with $\xi = r/u$, the normalizable solutions up to the first order of perturbation expansion, respectively, read

$$f^{(1)}(u, r) = C_3\, [u\, f(r)^2]^{\Delta_+}, \tag{9}$$

$$f^{(1)}(u, r) = \left[C_4 + C_5\, \ln\left(\frac{r}{u}\right)\right] \left(\frac{u}{r}\right)^{\Delta_+}. \tag{10}$$

But, here we use the ADM to obtain perturbative solutions in the form of series expansion. ADM [4] is a math method especially for solving NPDEs. In fact, for normalizable solutions for massive modes near the boundary ($u = 0$), we use

$$g_0(0, r) = g(0, r) - u \frac{\partial g(0, r)}{\partial u},$$
$$f(u \rightarrow 0, r) \equiv f(0, r) = f(r)\, u^{\Delta_+} \approx \bar{C}_{\Delta_+} \left(\frac{u}{r}\right)^{\Delta_+} \tag{11}$$

in the following iteration equation:

$$\blacksquare_4 g_{n+1} - \bar{M}^2\, g_{n+1} = \sum_{n=0}^{\infty} A_n, \qquad \blacksquare_4 \equiv \partial_i \partial_i + \partial_u \partial_u, \qquad \bar{M}^2 \equiv \frac{(2 + m^2)}{u^2}, \tag{12}$$

where the Adomian polynomials A_n, which come from nonlinear terms and act as perturbations, are as follows in the case of equation (8) - for details, see [3] and [1]

$$A_0 = 6 g_0^3, \qquad A_1 = 18\, g_0^2\, g_1, \qquad A_2 = 18\left(g_0^2\, g_2 + g_0\, g_1^2\right), \ldots . \tag{13}$$

As the same way, one can also write other iteration equations from equation (8) [4]. In this way, a series solution can be expanded to the nth order as $f^{(n)} = \sum_{n=0}^{n} f_n$.

Cont. Solving the Nonlinear Massive Equation by ADM

It is notable that equation (8) without the term $\bar{M}^2$ has the following exact solution:

$$\tilde{g}_0(u, \vec{u}) = \frac{2}{\sqrt{3}} \frac{b_0}{-b_0^2 + (u + a_0)^2 + (\vec{u} - \vec{u}_0)^2}, \tag{14}$$

where $\vec{u}_0 \equiv (b_1, b_2, b_3)$ with a_0 and b_i as the modules of the solution, and could be represented as the size and location of the instanton on the boundary, respectively. The latter solution has the behavior near the boundary as follows:

$$\tilde{g}_0(u \rightarrow 0, r) \equiv \tilde{g}_0(0, r) = \frac{2}{\sqrt{3}} \frac{b_0}{(a_0^2 - b_0^2 + r^2)} \left[1 - \frac{2\, a_0}{(a_0^2 - b_0^2 + r^2)}\, u\right]; \tag{15}$$

and this can be used, instead of $g(0, r)$ in (11), as the initial data in ADM.

Solutions of the Equation for $m^2 = 0$ and $m^2 = 40$ by ADM

In addition to the case including backreaction, where the massless mode appears, it is possible to realize such a state $m^2 = 0$ in the SW version of (3) with $C_1 = 1/3$ in probe limit (ignoring the backreaction). In the same way, the massive mode $m^2 = 40$ is realized in probe approximation in the WR version of (3) with $C_1 = 13$. In this case, using the equation (12), the initial conditions from (11) for $\Delta_+ = 3, 8$ and as a result, using the initial data $f_0 = (1 - \Delta_+)\, f(r)\, u^{\Delta_+}$, the solution of eq. (8) up to the third order in the perturbation series expansion for the massive and massless mode reads

$$f^{(3)}(u, r) = -6\, f(r)\, u^3 + \frac{1}{4} \bar{\nabla}^2 f(r)\, u^5 + O(u^7), \tag{16}$$

$$f^{(3)}(u, r) = -21\, f(r)\, u^8 + \frac{28}{135} \bar{\nabla}^2 f(r)\, u^{10} + O(u^{12}), \tag{17}$$

respectively, where $d^2/dr^2 + 2/(r\, dr) \equiv \bar{\nabla}^2$. In the same way and writing other iteration equations from (8), we obtain the following solutions for the massless and massive modes, respectively:

$$f^{(1)}(u, r) = \frac{9}{4}\left(\frac{\bar{b}_0\, u}{r^2 - \bar{b}_0^2}\right)^3, \quad f^{(1)}(u, r) = 3\left(\frac{\bar{c}_0\, u}{r^2}\right)^3, \tag{18}$$

$$f^{(1)}(u, r) \cong \frac{2}{100} \frac{u^8}{r^{13 + 5n_0}}. \tag{19}$$

Dual Symmetries and Solutions with AdS_4/CFT_3 correspondence

The original theory has the geometry $AdS_4 \times S^7/Z_k$ ($\rightarrow CP^3 \ltimes S^1/Z_k$), isometric symmetry $SO(3, 2)$ in the Minkowski space, the internal symmetry $SO(8) \equiv G$ ($\rightarrow SU(4) \times U(1) \equiv H$) and the supersymmetry $\mathcal{N} = 8 \rightarrow 6$. The 4-form ansatz (1) is actually attributed to the (anti)membranes (in the case where the background is WR, antimembranes and in the case where the field is SW, membranes) that are wraped around mixed internal and external directions and so break all SUSY's and parity, and the resulting theory is for anti-M2-branes. Likewise, with the resulting singlet (pseudo)scalars and equations in the EAdS_4 space that does not explicitly include any elements of the internal space, we actually have a consistent truncation. As a result, our bulk solutions preserve at least SO(4) symmetry, which by interpreting them as bubble solutions, the four parameters related to the breaking of scale and translational symmetries (i.e. a_0 and $\vec{u}_0$) are responsible to move the bubble around in the 4D bulk and size of the instanton.

To realize the resulting H-singlet (pseudo)scalars in the 11D SUGRA spectrum and SUSY breaking, we swap the three fundamental representations (reps) 8_v, 8_s, 8_c of $SO(8)$ for gravitino. As the same way, we focus on the $U(1) \times U(1)$ part of the original quiver gauge group and take just one scalar and one fermion (noting that the singlet (pseudo)scalar or fermion we consider could be taken from decomposing the eight (pseudo)scalars or fermions as $X^I \rightarrow (\Phi^n, \Phi, \bar{\Phi})$, with Φ representing either ψ or Y, $I, J \ldots = (1, \ldots 6, 7, 8) = (n, 7, 8)$ and $\Phi = \Phi^7 + i\, \Phi^8$, $\Phi^\dagger = \bar{\Phi}$, transforming in the rep $(6_0, 1_2, 1_{-2})$ under $G \rightarrow H$) in the boundary CS matter theory, and will find dual solutions.

On the other hand, for a bulk (pseudo)scalar with near the boundary behavior $f(u, \vec{u}) \approx \alpha(\vec{u})\, u^{\Delta_-} + \beta(\vec{u})\, u^{\Delta_+}$ (noting that for the massless and massive modes, only mode β is normalizable), we write the AdS/CFT dictionary as

$$\langle \mathcal{O}_{\Delta_+} \rangle_\alpha = -\frac{\delta W[\alpha]}{\delta \alpha} = \beta, \qquad \langle \mathcal{O}_{\Delta_-} \rangle_\beta = -\frac{\delta \widetilde{W}[\beta]}{\delta \beta} = \alpha,$$
$$\widetilde{W}[\beta] = -W[\alpha] - \int d^3\vec{u}\; \alpha(\vec{u})\, \beta(\vec{u}) \tag{20}$$

where $W[\alpha]$ ($\widetilde{W}[\beta]$) is the generating functional of the connected correlator of the operator $\mathcal{O}_{\Delta_+}$ ($\mathcal{O}_{\Delta_-}$) on the usual (dual) boundary CFT $_3$ with Δ_+ (Δ_-) quantization.

Dual Solutions in a Boundary 3D CS Matter Model

Considering a singlet scalar $Y = \varphi = h(r)\, I_N$ and a singlet fermion ψ (depending on the case) and $U(1)$ part of the gauge group, we write the boundary action as follows:

$$S^{(j)} = S_{CS}^{+} - \int d^3\vec{u} \left[tr(D_k Y^\dagger D^k Y) + tr(i\bar{\psi}\, \gamma^k D_k \psi) + \mathcal{W}_\Delta^{(j)}\right], \quad \mathcal{W}_\Delta^{(j)} = \alpha\, \mathcal{O}_{\Delta_+}^{(j)}, \tag{21}$$

where the integral of $\mathcal{W}_\Delta^{(j)}$ is W in (20) and represents the deformations (labeled by $j = a, b, \ldots, g$) that we do with different H-singlet operators; and the CS Lagrangian is

$$\mathcal{L}_{CS}^{+} = \frac{ik}{4\pi} \varepsilon^{ijk}\, tr\left(A_i^+ \partial_j A_k^+ + \frac{2i}{3} A_i^+ A_j^+ A_k^+\right); \tag{22}$$

and also $D_k \Phi = \partial_k \Phi + i A_k\, \Phi - i \Phi\, \hat{A}_k$ and $F_{ij} = \partial_i A_j - \partial_j A_i + i[A_i, A_j]$.

E-Mail: m.naghdi@ilam.ac.ir

Marginal Deformations and Dual Solutions for the Massless Mode

In this case, using both terms $S_{CS} + \hat{S}_{CS}$ instead of S_{CS}^{+} in action (21) and noting that $F_{ij}^- = 0$ and as a result $A_i^- = 0$ and also setting $\alpha = 1$ for simplicity, in addition to previously considered marginal operators, with $\mathcal{O}_3^{(g)} = tr(\varphi\bar{\varphi})\, tr(\psi\bar{\psi})^{1/2}\, \varepsilon^{kij} \varepsilon_{ij}\, A_k^+$, the resulting equations for scalar ($\bar{\varphi}$), fermion ($\bar{\psi}$) and unit (A_k^+) become

$$\partial_k \partial^k \varphi - \varphi\, tr(\psi\bar{\psi})^{1/2}\, \varepsilon^{kij} \varepsilon_{ij}\, A_k^+ = 0, \tag{23}$$

$$i\, \gamma^k \partial_k \psi + \frac{\psi}{2}\, tr(\psi\bar{\psi})^{-1/2}\, tr(\varphi\bar{\varphi}) \varepsilon^{kij} \varepsilon_{ij}\, A_k^+ = 0, \tag{24}$$

$$\frac{ik}{4\pi} \varepsilon^{kij} F_{ij}^+ - tr(\varphi\bar{\varphi})\, tr(\psi\bar{\psi})^{\frac{1}{2}}\, \varepsilon^{kij} \varepsilon_{ij} + 2\, \bar{\psi}\, \gamma^k\, \psi + i\left[\varphi(\partial^k \bar{\varphi}) - (\partial^k \varphi)\bar{\varphi}\right] = 0, \tag{25}$$

which in the last equation we have used $\varphi \neq \bar{\varphi} = \varphi^\dagger$ which is allowed in Euclidean space; and $\gamma^k = (\sigma_2, \sigma_1, \sigma_3)$ are Euclidean gamma matrices. From the solving of equations (23), (24) and (25) together, considering $\varphi = h(r)\, I_N$, $\varphi^\dagger = a_5\, I_N$, solution is

$$\psi = a_3 \frac{a + i(\vec{u} - \vec{u}_0).\vec{\gamma}}{[a^2 + (\vec{u} - \vec{u}_0)^2]^{\varsigma = 3/2}} \binom{1}{0}, \qquad h = \frac{3}{4}\left(\frac{a_6}{a^2 + (\vec{u} - \vec{u}_0)^2}\right), \tag{26-7}$$

$$A_k^+ = \varepsilon_{kij}\, \varepsilon^{ij} A^+(r), \qquad A^+ = \frac{3}{4} \frac{a}{a^2 + (\vec{u} - \vec{u}_0)^2}, \tag{28}$$

where $a_0, a_1, a_2, \ldots$ are boundary constants and $A^+(r)$ is a scalar function on the boundary.

As a result, we have

$$\langle \mathcal{O}_3^{(g)} \rangle_\alpha = \frac{9}{16} \frac{a\, a_3 a_5 a_6}{[a^2 + (\vec{u} - \vec{u}_0)^2]^3}, \tag{29}$$

which can be matched with the bulk solution on the LHS of (18); Or make it correspond to the solution (7) for $\Delta_+ = 3$, in which case the boundary solution can be considered as an instanton at the conformal point $u = a$. In the same way, the correction to the corresponding action can be calculated based on the above solutions, which results in:

$$S_3^{(g)} = -\frac{1}{2} \int \mathcal{O}_3^{(g)}\, d^3\vec{u} = -\frac{9\pi}{2} \int_0^\infty \frac{a\, a_3 a_5 a_6\, r^2}{(a^2 + r^2)^3}\, dr \Rightarrow \tilde{S}_{modi.}^{(g)} = -\frac{9}{32} \pi^2 a; \tag{30}$$

(in the last step, for simplicity, we set all constant parameters equal) and this is a finite value that represents an instanton with size $a \geq 0$ (in the limit $a \rightarrow 0$, a small instanton) in the center ($\vec{u}_0 = 0$) of a three-sphere with radius r is at infinity (S_∞^3).

Irrelevant Deformations and Dual Solutions for the Massive Mode

For the Higgs-like mode $m^2 = 40$, we can perform irrelevant deformations corresponding to the Dirichlet boundary condition with several $\Delta_+ = 8$ operators [4] such as

$$\mathcal{O}_8^{(a)} = tr(\psi\bar{\psi})^4, \qquad \mathcal{O}_8^{(b)} = tr(\varphi\bar{\varphi})^4\, tr(\psi\bar{\psi})^2,$$
$$\mathcal{O}_8^{(e)} = tr(\psi\bar{\psi})\, tr(\varphi\bar{\varphi})^3\, F^+ \wedge A^+, \qquad \mathcal{O}_8^{(f)} = tr(\varphi\bar{\varphi})^6\, \varepsilon^{ij}\, F_{ij}^+. \tag{31}$$

For example, with $\mathcal{O}_8^{(f)}$, leaving aside the fermionic part of the action and performing the deformation according to (21), the equations of motion for $\bar{\varphi}$ and A_i^+ are as follows:

$$\partial_k \partial^k \varphi - 6\, \alpha\, \varphi\, tr(\varphi\bar{\varphi})^5\, \varepsilon^{ij}\, F_{ij}^+ = 0, \qquad \frac{ik}{4\pi} \varepsilon^{kij} F_{ij}^+ + i\left[\varphi(\partial^k \bar{\varphi}) - (\partial^k \varphi)\bar{\varphi}\right] = 0. \tag{32-3}$$

In the case with $\varphi = \bar{\varphi} = h(r)\, I_N$, the solution of the gauge part can be

$$F^+ \equiv \varepsilon^{ij}\, F_{ij}^+ = \left(\frac{a}{a^2 + (\vec{u} - \vec{u}_0)^2}\right)^2, \tag{34}$$

which, with a non-zero finite a, it satisfies the condition $F^+(r \rightarrow \infty) \rightarrow 0$; And then considering $F^+ = -h^4$ and $\alpha = tr(\varphi\bar{\varphi})^{-5}$, the equation (32) becomes:

$$\partial_k \partial^k h + 6\, h^5 = 0 \Rightarrow h = \left(\frac{1}{2}\right)^{1/4} \left(\frac{a}{a^2 + (\vec{u} - \vec{u}_0)^2}\right)^{1/2}; \tag{35}$$

And as a result, $\langle \mathcal{O}_8^{(f)} \rangle_\alpha = a_9 \left(\frac{a}{a^2 + (\vec{u} - \vec{u}_0)^2}\right)^8$ (36), with $a_9 = 1/8$, which in the limit of $a \rightarrow 0$ and $r \rightarrow \infty$ coincides structurally with the bulk near the boundary solutions (17) and (19). Also, this boundary solution can be considered as an instanton that sits at the conformal point of $u = a$, which in this case matches with the bulk solution (7) with $\Delta_+ = 8$.
Moreover, the (finite) value of the action based on the recent solution reads

$$S_8^{(f)} = 5 \int \mathcal{W}_8^{(f)}\, d^3\vec{u} = \frac{20\pi}{\sqrt{2}} \int_0^\infty \frac{a^3\, r^2}{(a^2 + r^2)^4}\, dr \Rightarrow \tilde{S}_{modi.}^{(f)} = \frac{5\pi^2}{4\sqrt{2}}. \tag{37}$$

More Points

According to the general form of solutions (26) for fermion (ψ), (28) for gauge field (A^+) (or (34) for F^+) and (35) for scalar (φ), we may have a type of Bose-Fermi duality in the limit of solutions and correspondence as $tr(\psi\bar{\psi}) \sim tr(\varphi\bar{\varphi})^2 \sim F^+$ and $\psi \sim A^+$.
As a result, the potentials attributed to the boundary deformations will be unbounded from below, which accept Fubini-like instantons. In fact, the scalar potential from equation (3) is $V(f) = \frac{m^2}{2} f^2 + \frac{\lambda}{4} f^4$, which with $m^2 < 0$ is a double-well potential that accepts instanton solutions or Coleman-Di Lucia bonuses. The corresponding bulk solutions can actually describe vacuum decay or quantum tunneling and correspond to the growth and expansion of true vacuum bubbles in the background of the bulk false vacuum; and the final fate of such bubbles will be a big collapse or crunch in the anti-de sitter space.

References

Hope you Enjoy!

[1] **M. Naghdi**, *Chin. Phys. C* **48**, 043104 (2024), [arXiv:2311.11671 [hep-th]].
[2] **O. Aharony**, **O. Bergman**, **D. L. Jafferis** and **J. Maldacena**, *JHEP* 0810, **091** (2008), [arXiv:0806.1218 [hep-th]].
[3] **M. Naghdi**, *Eur. Phys. J. Plus* **138**, 45 (2023), [arXiv:2002.06547 [hep-th]].
[4] **M. Naghdi**, *Eur. Phys. J. Plus* **138**, 300 (2023), [arXiv:2005.00358 [hep-th]].
[5] **G. Adomian**, "Solving frontier problems of physics: The decomposition method", *Springer*, 1st Edition (1994).
[6] **M. Naghdi**, "A Consistent Truncation of 11D Supergravity, (pseudo)Scalars in AdS_4 Space and Exact Dual Solutions in the Boundary 3D Field Theory", [**Preprint**].