# Exotic interstellar polyatomic $H_3^+$ mass spectrum

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## Exotic interstellar $H_3^+$

Exotic interstellar polyatomic $\boldsymbol{H_3^+}$ is the simplest polyatomic bound state [1]. It exists in various interstellar environments, near the Galactic center, especially close to the star-forming regions that exhibit a significant presence of extensive and dispersed warm gas where exotic contributes to the interactions and dynamics of the medium with temperatures on the order of a few hundred to a few thousand Kelvin. This exotic bound state plays a crucial role in astrophysics, spectroscopy and observational astronomy, molecular cloud dynamics, chemical processes, and physics of the interstellar medium. By studying the mass spectrum and energy eigenvalues of exotic $\boldsymbol{H_3^+}$, for the ground and orbit excited states under the relativistic conditions and quantum field theory principles, we can gain insights into the physical and chemical properties of interstellar regions, due to provide valuable information about the conditions and processes occurring in the spaces between stars, planetary systems and hot mediums. In this theoretical research, we explain the analytic method for determination of the mass spectrum of the exotic interstellar polyatomic $\boldsymbol{H_3^+}$ as a bounded system under the strong Coulombic field within the relativistic corrections on the mass of bounded particles. Exotic $\boldsymbol{H_3^+}$, is a simple composed of the specific form of two hydrogen atoms and one hydrogen cation named tri-hydrogen cation. It is formed when a hydrogen atom loses one electron, resulting in a positively charged ion, and consists of three protons and two shared electrons. This exotic bound state is an example of a triatomic ion state and is the simplest polyatomic ion, while we can present interstellar $\boldsymbol{H_3^+}$ due to the main cores (i.e., proton) as a semi-hadronic-molecular bound state system. As we know, the mass spectrum of the bound state $\boldsymbol{H_3^+}$ can be determined with good precision in the framework of nonrelativistic conditions when a good selection of the potential is made. However, the nonrelativistic Schrödinger equation which gives a mathematically correct description of $\boldsymbol{H_3^+}$ is rare sufficient for the description of new astrophysical observations and data obtained in exotic particle physics, hence it is necessary to include the relativistic correction on the properties and behavior of interstellar $\boldsymbol{H_3^+}$ that is the main goal of this theoretical research.

## Bound states in the functional approach

A good technique for considering the exotic interstellar $\boldsymbol{H_3^+}$ relativistic states in the strong electrostatics and hot medium are the asymptotic behavior of five-point Green's function at semi-high energy limit, which is acquired in the exponentiated model using the large asymptotic limit of $x \to \infty$ from the Feynman functional path integral (FFPI) in the external vector field [2-6]. The technique is applied to the basic fundamental model of quantum and relativistic field theory. In this approach the properties of the exotic interstellar $\boldsymbol{H_3^+}$ vacuum polarization function in closed-loop through the FFPI is offered and defined based on the quantum field theory approach [6]. Therefore, the mass spectrum is determined by $\boldsymbol{H_3^+}$ vacuum polarization function (statistical correlation) and the Green's function [2-4]. The Green's function is described in FFPI form and then the mass spectrum should be explained in relativistic quantum theory as following relations [8]

$$M = \frac{1}{2}\min_{\mu_1,\dots,\mu_5}\left(\sum_{i=1}^{5}\frac{m_i^2}{\mu_i} + \sum_{i=1}^{5}\mu_i + 2E_n(\mu_1,\dots,\mu_5)\right) \tag{1}$$

and

$$\mu_i - \frac{m_i^2}{\mu_i} + 2\mu_i\min_{\mu_1,\dots,\mu_5} E(\mu_1,\dots,\mu_5) = 0, \text{for} i = 1,\dots,5 \tag{2}$$

where $\mu_i$ is the mass of the constituent particle that is equivalent to the relativistic mass of the particle in the moving system at relativistic velocity, $m_i$ is the rest mass of constituted particles at the free state and $E_n$ is the energy eigenvalues in the main quantum states $n = 1,2,\cdots$. The theoretical method of FFPI with the additional proof and analysis is completely described in reference [2-5].

## Schrödinger equation under the relativistic correction

The behavior of the interstellar $\boldsymbol{H_3^+}$ and the relativistic correction to the Hamiltonian in a strongly interacting charged environment at the finite temperature on the order of a few hundred to a few thousand Kelvin is presented. As we know, in the interstellar medium where the main nuclide of the star is created, we can consider that particles and systems are near the high energy interactions. Hence, we use the high-energy approximation $\sqrt{m_i^2 + \hat{p}^2} \approx \min_{\mu_i}\frac{1}{2}\left(\mu_i + \frac{m_i^2+\hat{p}^2}{\mu_i}\right)$, for the relativistic energy condition of each particle. $\mu_i$ is the relativistic mass of particle $i$, with the rest mass $m_i$. Now, based on the modified radial Schrödinger equation for Coulombic potential with the constant of interaction $\alpha_s$,

$$\left(\sum_{i=1}^{5}\sqrt{m_i^2 + \hat{p}^2} + \sum_{\substack{i=1.\\ i<j.\\ i\neq j}}^{5}\frac{e_i e_j}{|r_i - r_j|}\alpha_s\right)R(r) = E_n R(r) \tag{3}$$

using the projective unitary and Wick ordering methods we can solve equation (3). These models describe canonical variables in the form of the creation and the annihilation operators [2-8]

$$\hat{q} = \frac{1}{\sqrt{2m\omega}}(\hat{a} + \hat{a}^\dagger),\quad \hat{p} = i\sqrt{\frac{m\omega}{2}}(\hat{a}^\dagger - \hat{a})$$

where

$$\hat{a}^\dagger = \frac{1}{\sqrt{2m\omega}}(m\omega\hat{q} - i\hat{p}) = \frac{1}{\sqrt{2m\omega}}\left(m\omega\hat{q} - \frac{\partial}{\partial\hat{q}}\right)$$
$$\hat{a} = \frac{1}{\sqrt{2m\omega}}(m\omega\hat{q} + i\hat{p}) = \frac{1}{\sqrt{2m\omega}}\left(m\omega\hat{q} + \frac{\partial}{\partial\hat{q}}\right)$$

Then the canonical variables represented in the quadratic form to explain potential interaction, and momentum as follows

$$\hat{q}^2 \cong \frac{d}{2\omega} + :\hat{q}^2: \qquad\qquad \hat{p}_q^2 = \frac{d}{2}\omega + :\hat{p}^2:$$

where:∗: is the condition in the Wick ordering method and according to the projective unitary conditions the interaction Hamiltonian with the minimum energy of the bound state can be defined in the new axillary symplectic space with dimension $d$, for details [8]. By introducing an auxiliary symplectic space as a powerful mathematical tool and technique, one can provide a clearer understanding of the dynamics of bound states. For this reason, we identify canonical operators by changing $r = q^2$. The radial Laplacian for these intertwined spaces can be modified by

$$\frac{d^2}{dr^2} + \frac{N-1}{r}\frac{d}{dr} \to \Delta_q = \frac{d^2}{dq^2} + \frac{\mathcal{D}-1}{q}\frac{d}{dq}$$

Therefore, using equation (3) and definition of equation (1) and (3), the mass spectrum of interstellar $\boldsymbol{H_3^+}$ the bound state can be presented using $m_1 = m_2 = m_3$, $\mu_1 = \mu_2 = \mu_3$ and $m_4 = m_5$, $\mu_4 = \mu_5$ in the natural unite ($\hbar = c = 1$) as follows

$$M = 3\mu_1 + 2\mu_4 + \sum_{i=1}^{4} M_i\frac{dE_n}{dM_i} + +E_n(M_2, M_3, M_4) \tag{4}$$

where

$$\frac{1}{M_2} = \frac{1}{\mu_1} + \frac{1}{\mu_2}$$
$$\frac{1}{M_3} = \frac{1}{\mu_1+\mu_2} + \frac{1}{\mu_3}$$
$$\frac{1}{M_4} = \frac{1}{\mu_1+\mu_2+\mu_3} + \frac{1}{\mu_4}$$
$$\frac{1}{M_5} = \frac{1}{\mu_1+\mu_2+\mu_3+\mu_4} + \frac{1}{\mu_5}$$

The constituent mass is defined by

$$1 - \frac{m_1^2}{\mu_1} + 2\frac{M_2^2}{\mu_1^2}\frac{dE_n}{dM_2} + 2\frac{M_2^2}{(\mu_1+\mu_2)^2}\frac{dE_n}{dM_2} + \cdots + 2\frac{M_4^2}{(\mu_1+\mu_2+\mu_3+\mu_4)^2}\frac{dE_n}{dM_4} = 0$$
$$\vdots$$
$$1 - \frac{m_4^2}{\mu_4} + 2\frac{M_4^2}{\mu_4^2}\frac{dE_n}{dM_4} + 2\frac{M_5^2}{(\mu_1+\mu_2+\mu_3+\mu_4)^2}\frac{dE_n}{dM_5} = 0$$
$$1 - \frac{m_5^2}{5} + 2\frac{M_5^2}{\mu_5^2}\frac{dE_n}{dM_5} = 0$$

Thus, we explained the mass spectrum and constituent mass of Exotic interstellar polyatomic $\boldsymbol{H_3^+}$ bound state taking into account relativistic correction on mass.

## References

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