Splitting Hamiltonian of bound state by nonperturbative form of relativistic and spins interaction

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ABSTRACT

A nonperturbative Hamiltonian of bound state refers to a Hamiltonian operator that cannot be treated using perturbation theory. Perturbation theory is a mathematical approach used to solve problems when the Hamiltonian can be separated into a solvable unperturbed part and a perturbation that is small compared to the unperturbed part. Solving nonperturbative Hamiltonians typically requires the use of more advanced techniques, such as numerical methods or approximation schemes specifically designed for nonperturbative systems, which is a great deal of interest in nuclear and particle physics. We calculate the mass spectrum and constituent mass of particles in a bound state using the Bethe–Salpeter equation with $U(r) = U_v(r) + U_s(r)$ potential of interaction at high energy and relativistic limits. Therefore, the most essential issue in theoretical particle research is to explain the Einsteinian adjustment of higher bound states, in order to determine the characteristics of relativistic effects within the potential interaction and kinetic energy. We present the method based on quantum field theory and Feynman path integral to calculate the mass spectra of hadrons. As we know, the long-range behavior of the propagator function of the related currents with the specific quantum numbers can determine the mass spectra of hadronic bound states. The presentation of the propagator in quantum field theory as a functional integral allows us to average over the external field. This approach is very close to the Feynman functional path integral in quantum physics, where relativistic effects are not considered. By the side of the path integral, the Feynman diagram determines the interaction potential within the exchange of the mass and the field. We explain the related current of charged particles in the hadronic state and represent the propagator in the form of the corresponding current by averaging over the field **A** for two bounded particles. This defines the kernel function of two charged particles with the rest masses. Then we can determine the two-point function by averaging over the field $\Pi(r - r') = \langle G_{m_c} G'_{m_{\bar{c}}} \rangle_{\boldsymbol{A}}$. By the variational method, the two-point function presents in the form of path integral, which is like Feynman's functional in non-relativistic quantum physics. The two-point function and the propagator at the limited distant $x \to 0$ present the Feynman path integral for the motion of particles with masses μ_1, μ_2 in the quantum theory with the $U_{i,j}$ potential interactions. The total potential interaction within the relativistic corrections reads

$$W_{i,j} = \frac{g^2 i^{i+j}}{2} \iint d\tau_1 d\tau_2 Z^i(\tau_1) G\big(Z^i(\tau_1) - Z^j(\tau_2)\big) Z^j(\tau_2) \quad (1)$$

where the functional integral is over the 4-dimensional spacetime, $G\big(Z^i - Z^j\big)$is the propagator of the field **A**, $U_{1,1}, U_{2,2}$ is the self-energy of interacted particles, and $U_{1,2}$ is the interaction of particles with the field **A** and we choose the dependence of Euclidean time $r^{(0)}$ on τ (proper time) for the composite particle as follows: $r^{(0)} = c(\tau_1 - \tau_2)u = c\tau u.$ Now let's proceed to defining the structure of the interaction Hamiltonian. The interaction between composite particles occurs through the exchange of gauge fields with the relativistic velocity of composite particles $v(\tau) =$

$\frac{\partial\, r(\tau)}{\partial \tau}$, so we will express the propagator in the standard form $G(q^2+\frac{s^2}{c^2}) \approx \int_0^{\infty} d\eta e^{-\eta\left(q^2+\frac{s^2}{c^2}\right)}$, and after integrating over dq, we have the following for the interaction potential:

$$U_{i,j} = \frac{g^2 i^{i+j}}{4\pi} \iint d\tau_1 d\tau_2 \frac{\delta(\tau_1-\tau_2)}{|r_i(\tau_1)-r_j(\tau_2)|} + \frac{g^2 i^{i+j}}{4\pi} \sum_k \frac{i^k}{(2k)!c^{2k}} \int_0^t d\tau \frac{\partial^{2k}}{\partial \tau^{2k}} \left| r_i(\tau_1) - r_j(\tau_2) \right|^{2k-1} \quad (2)$$

Now, we consider the case where the relative velocity is constant $v(\tau) = \frac{\partial\, r(\tau)}{\partial \tau} = 0$, in this case, the contribution of the nonperturbative correction to the interaction Hamiltonian is represented as a sum in equation (2) $I = \sum_k \frac{i^k}{(2k)!c^{2k}} \frac{\partial^{2k}}{\partial \tau^{2k}} |r(\tau)|^{2k-1}$, in this approximation, for different values of k, we obtain $I = \frac{\hat{\ell}^2}{r^3} + \frac{9\hat{\ell}^4}{r^5} + \cdots + \frac{\hat{\ell}^{2n}}{r^{2n+1}} \prod_{i=1}^{n} (2i-1)^2$, where $\hat{\ell} = [\vec{r}.\ \vec{v}]$, $\hat{\ell}$ - is orbit momentum operator. Then, after doing a series of mathematical rewrites, we obtained a nonperturbative correction to the interaction Hamiltonian that is associated with the relativistic nature of the system. In particular, our system consists of hadronic particles moving relative to each other with a constant velocity. The obtained correction to the Hamiltonian, related to the relativistic nature of the interaction, vanishes in the nonrelativistic limit as c goes to infinity. In deriving the nonperturbative correction, we assumed a simple relationship between Euclidean and proper times for the constituent particles. In principle, in our approach, it is possible to consider any dependence of Euclidean time on τ. Hence, we will define the nonperturbative corrections to the Hamiltonian $H = H_0 + H_{nonper.} = H_0 + H^0_{nonper.} + H^{LS}_{nonper.} + H^{TT}_{nonper.}$ As follows

$$H^0_{nonper.} = U_v(r) \left[\frac{1}{\sqrt{1+\frac{\ell(\ell+1)}{r^2 c^2}}} - 1 \right] \quad (3)$$

Now, we define the nonperturbative corrections if the relative velocity is $v(\tau) = \frac{\partial\, r(\tau)}{\partial \tau} \neq 0$, $\dot{v}(\tau) = \frac{\partial\, v(\tau)}{\partial \tau} \neq 0, \ddot{v}(\tau) = \frac{\partial\, v(\tau)}{\partial \tau} = 0$. In this case, the contribution of the nonperturbative correction to the interaction Hamiltonian is represented based on the contribution with Thomas precession. Thomas precession is a relativistic effect that arises in the context of bound states, where particles are confined within a potential interaction. It occurs due to the combined effects of the relativistic motion of the particles and their intrinsic magnetic moments. Thomas precession causes a precession or rotation of the spin or angular momentum of the particles around the direction of their motion. This additional precession arises as a consequence of the relativistic time dilation and length contraction effects. So we define all terms based on quantum electrodynamics when charged particles undergo accelerated motion, quantities $\frac{\partial \hat{\ell}}{\partial \tau}, \frac{\partial^2 \hat{\ell}}{\partial \tau^2}$ and $\frac{1}{cr}\frac{\partial \hat{\ell}}{\partial \tau}$ are related to the spin precession. This means that the accelerated motion of the charged particles can induce a precession of their spin, which in turn affects their dynamics and can lead to the emission of electromagnetic radiation. These quantities are associated with spin precession and read

$$\frac{\partial \hat{\ell}}{\partial \tau} = \frac{\partial [\vec{r}.\ \vec{v}]}{\partial \tau} = \left[\vec{r}.\ \vec{\dot{v}}\right], \quad \frac{1}{cr}\frac{\partial \hat{\ell}}{\partial \tau} = \frac{1}{c}\left[\vec{n}.\ \vec{\dot{r}}\right], \quad \frac{\partial^2 \hat{\ell}}{\partial \tau^2} = \frac{\partial [\vec{r}.\ \vec{\dot{v}}]}{\partial \tau} = \left[\vec{v}.\ \vec{\dot{v}}\right] \quad (4)$$

where $\vec{r}$ is the distance between the particles, $v = (\vec{n}.\vec{v})$- is the relative velocity of particles. Then the interaction Hamiltonian associated with Thomas precession taking into account the above equations read

$$H^{LS}_{nonper.} = -\frac{U_v(r)}{4}\frac{\hat{\ell}[\vec{v}.\dot{\vec{v}}]}{v^4}\left[3\ln(1+v^2) - v^2\frac{1-v^2}{1+v^2}\right] \quad (5)$$

and

$$H^{TT}_{nonper.} = -\frac{U_v(r)}{256}\frac{r^2}{v^6}\left(\frac{1}{\sqrt{1-v^2}} - 1\right)\left[\vec{v}.\ \dot{\vec{v}}\right]^2\left[1 - \sqrt{1+4v^2} + 8v^2 - 8v^4 + \frac{8}{3}v^6\right] \quad (6)$$

Parameter $\Omega = \frac{[\vec{v}.\dot{\vec{v}}]}{v^2}$ – is the frequency of Thomas precession.

References

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