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Acoustic vortex in media with screw dislocation using Katanaev-Volovich approach

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Abstract

We study acoustic vortex in media with screw dislocation using Katanaev-Volovich theory of defects. It is shown that the screw dislocation affects the beam's orbital angular momentum. This change of acoustic vortex strength is a manifestation of topological Dirac phase.

Acoustic vortex as the classical counterpart of optical vortex has been studied and generated, recently. Since acoustic vortices can transfer orbital angular momentum to particles, they can be applied to particle trapping in acoustic tweezers and remote controlling. In addition acoustic vortices, potentially, can be used in sonar experiments. In this letter, acoustic vortex in media with screw dislocation is studied using the Katanaev-Volovich theory of defects. The motivation for studying defects is that they usually exist in crystalline solids and have strong effect on their physical properties.

In the presence of defects, we are confronting with complicated boundary conditions. This difficulty persuades physicists to introduce new approaches such as Katanaev-Volovich theory of defects in solids [1,2]. Katanaev-Volovich theory is a geometrical approach based on the isomorphism existing between the theory of defects in solids and three-dimensional gravity. In this formalism, elastic deformation which is introduced in the medium by defects is replaced by a non-Euclidean metric.

One of the defects, which we are interested in, is a screw dislocation. In a screw dislocation the Burgers vector is parallel to the dislocation line. This kind of defect, which corresponds to a singular torsion along the defect line, is described by the following metric [1,4]

$$ds^2 = g_{ij}dx^i dx^j = (dz + \beta d\phi)^2 + d\rho^2 + \rho^2 d\phi^2, \qquad (1)$$

where the parameter β is related to the Burgers vector, $\boldsymbol{b}$, by $\beta = \frac{b}{2\pi}$ and the screw dislocation line is oriented along the z-axis of the cylindrical coordinates (ρ, φ, z).

According to the Katanaev-Volovich approach, the dynamic of displacement vector field $\boldsymbol{U}(\boldsymbol{x}, t)$ in an elastic medium with defect is governed by

$$\partial_t^2 U^i = \frac{\mu}{\rho}\tilde{\nabla}^2 U^i + \frac{(\lambda + \mu)}{\rho}\tilde{\nabla}^i \tilde{\nabla}_j U^j. \qquad (2)$$

where λ and μ are the Lame coefficients. ρ is the density of the medium and $\tilde{\nabla}$ is the generalized covariant derivative. As far as we know from vector analysis, it is always possible to express a vector as the sum of the curl of a vector and the gradient of a scalar. So equation (2) decomposes into two independent equations for transverse and longitudinal parts of the displacement vector field,

$$\frac{1}{v_T^2}\partial_t^2 U^{Ti} - \tilde{\nabla}^2 U^{Ti} = 0, \qquad \frac{1}{v_L^2}\partial_t^2 U^{Li} - \tilde{\nabla}^2 U^{Li} = 0, \tag{3}$$

where

$$v_T^2 = \frac{\mu}{\rho}, \qquad v_L^2 = \frac{(\lambda + 2\mu)}{\rho},$$

are the speeds of transverse and longitudinal parts in the medium. $\tilde{\nabla}^2$ in (3) is the Laplace-Beltrami operator which is given by $\tilde{\nabla}^2 = \frac{1}{\sqrt{g}}\partial_i(g^{ij}\sqrt{g}\partial_j)$, where g is the determinant of the metric tensor g_{ij} and $g^{ij} = (g_{ij})^{-1}$ is its inverse. Here we are interested in the longitudinal mode but similar results can be deduced for the transverse mode, too.

Using the metric (1) for screw dislocation, the longitudinal part of the elastic wave in (2) takes the form

$$\{\frac{1}{\rho}\partial_\rho(\rho\partial_\rho) + \frac{1}{\rho^2}(\partial_\varphi - \beta\partial_z)^2 + \partial_z^2\}U^{Li}(\rho,\varphi,z,t) = \frac{1}{v_L^2}\partial_t^2 U^{Li}(\rho,\varphi,z,t).$$

Considering a monochromatic paraxial wave, $U^{Li}(\rho,\varphi,z,t) = e^{-i\omega t}e^{ikz}u^{Li}(\rho,\varphi)$, yields to the following equation for longitudinal elastic wave

$$\{\frac{1}{\rho}\partial_\rho(\rho\partial_\rho) + \frac{1}{\rho^2}(\partial_\varphi - i\beta k)^2 - k^2\}u^{Li}(\rho,\varphi) = -\frac{\omega^2}{v_L^2}u^{Li}(\rho,\varphi). \tag{4}$$

Introducing a momentum operator as $\boldsymbol{P} = -i\nabla$ converts (4) into a time-independent Schrödinger-like equation with a gauge potential $\boldsymbol{A} = \frac{k\beta}{\rho}\hat{\boldsymbol{e}}_\varphi$ as

$$(\boldsymbol{P} - \boldsymbol{A})^2 u^{Li}(\rho,\varphi) = \frac{\omega^2}{v_L^2}u^{Li}(\rho,\varphi). \tag{5}$$

Since the gauge is curl free, $\nabla \times \boldsymbol{A} = 0$, the perfect analogy is seen between acoustic waves in media with screw dislocation and the Aharanov-Bohm effect. According to this correspondence, Dirac phase factor method [3] can be used here. Thus, the solution of the wave equation (5) has the following property

$$u^{Li}(\rho,\varphi) = \exp\{i\int_C \boldsymbol{A}\cdot d\boldsymbol{r}\}u_0^{Li}(\rho,\varphi), \tag{6}$$

where $u_0^{Ti}(\rho,\varphi)$ is the solution of the defect free case, $A = 0$, and C is the beam trajectory. So we will have

$$u^{Li}(\rho,\varphi) = e^{i\int_0^{\varphi} k\beta d\varphi}u_0^{Li}(\rho,\varphi). \tag{7}$$

This means that $u^{Li}(\rho,\varphi)$ differs from $u_0^{Li}(\rho,\varphi)$ just in a phase factor $e^{i\gamma}$, $\gamma = \int_0^{\varphi} k\,\beta d\varphi$, which is called Dirac phase factor. In the defect free case, $\beta = 0$, the solution of the wave equation (5) can be easily found as, $u_0^{Li}(\rho,\varphi) = R(\rho)e^{il\varphi}$, where $R(\rho)$ is the radial solution of the Helmholtz equation and $e^{il\varphi}$ represents acoustic vortex carrying the angular momentum l, acoustic vortex strength, along the paraxial axis. So the solution of the wave equation in the presence of screw dislocation is shown as

$$u^{Li}(\rho,\varphi) = R(\rho)e^{i(l+\beta k)\varphi}.$$

Therefore, the screw dislocation results in the change of the acoustic vortex strength from l to $l + \beta k$. This change, due to Dirac phase, is proportional to the magnitude of Burgers vector.

Conclusion

In this letter we studied the effect of the screw dislocation on an acoustic wave using the Katanaev-Volovich theory of defects. It was shown that the screw dislocation changes the acoustic vortex strength which is a manifestation of the topological Dirac phase.

References

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