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Transition in electric and magnetic charge of a dyon

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Abstract

This study investigates the physical properties of an electrically charged monopole-antimonopole chain (MAC) solution of Higher energy branch with ϕ - winding number of $n=4$, which undergoes two geometrical transition for increasing value of Higgs self-coupling constant, λ within the interval of the study. According this study, an electric and a magnetic charge transformation occurs for the pole which is located at the origin during second geometrical transition.

1. Introduction

Multimonopole numerical solutions in the SU(2) Yang-Mills-Higgs (YMH) theory with topological charges greater than one and an axial symmetry including monopole-antimonopole pair (MAP) and monopole-antimonopole chain (MAC) are introduced in the [1]. For MAP configurations, the Higgs field vanishes at two isolated symmetric points with respect to the origin on the symmetry axis (z- axis) while for MAC solutions the number of these isolated points is more than two. These solutions also include another kind of configuration which is known as vortex-ring configuration. For vortex-rings, The Higgs field vanishes on the rings centered on the symmetry axis. These rings are always located symmetrically with respect to x-y plane.

An electrically charged magnetic monopole is known as a dyon. Dyons with axial symmetry and electric charge parameter $0 \le \eta \le 1$ were first introduced by Hartmann *et al*. [2]. MAP solutions with critical electric charges are studied in some more detail in Ref. [3]. Ref. [3] covers only the values of $\lambda = 0$ and 1 for Higgs self-coupling constant.

Further studies show that at higher values of Higgs self-coupling constant, λ, beside the fundamental solution, other bifurcating solutions appear at the critical point of bifurcation [4], [5]. Each bifurcation includes a pair of new solutions. At some critical values of λ, the number of poles on the symmetry axis or the number of vortex-rings changes. These significant geometrical changes are called transitions between configurations.

In this study we concentrate on the higher energy bifurcating solution of electrically charged three-pole MAC solution with ϕ-winding number of $n = 4$ when the electric charge parameter is

η=0.5. Two geometrical transitions occur along this specific solution. This study shows that during the second transition, the sign of electric and magnetic charge of the dyon which is located at the origin changes.

2. The structure of electric and magnetic fields

For 3+1 dimensional YMH theory with a metric contract of $g_{\mu\nu}=(-+++)$, equations of motion are given by

$$D^{\mu}F^{a}_{\mu\nu}=\partial^{\mu}F^{a}_{\mu\nu}+\epsilon^{abc}A^{b\mu}F^{c}_{\mu\nu}=\epsilon^{abc}\Phi^{b}D_{\nu}\Phi^{c},$$

$$D^{\mu}D_{\mu}\Phi^{a}=\lambda\Phi^{a}(\Phi^{b}\Phi^{b}-\xi^{2}). \qquad (1)$$

where ξ is the vacuum expectation value of the Higgs field. Upon symmetry breaking, using the electromagnetic field tensor proposed by 't Hooft, Abelian electric field is $E_i=F_{i0}=\partial_i A_0-\partial_0 A_i$. and magnetic field is $B_i=-\frac{1}{2}\epsilon_{ijk}F_{jk}=\frac{1}{2}\epsilon_{ijk}\epsilon_{abc}\hat{\Phi}^{a}\partial^{j}\hat{\Phi}^{b}\partial^{k}\hat{\Phi}^{c}-\frac{1}{2}\epsilon_{ijk}\partial_{j}A_{k}$.

The *Ansatz* used for solving axially symmetric dyon solutions is

$$A^{a}_{i}=-\frac{1}{r}\psi_1(r,\theta)\hat{n}^{a}_{\phi}\hat{\theta}_i+\frac{1}{r}\psi_2(r,\theta)\hat{n}^{a}_{\theta}\hat{\phi}_i+\frac{1}{r}R_1(r,\theta)\hat{n}^{a}_{\phi}\hat{r}_i-\frac{1}{r}R_2(r,\theta)\hat{n}^{a}_{r}\hat{\phi}_i,$$

$$A^{a}_{0}=\tau_1(r,\theta)\hat{n}^{a}_{r}+\tau_2(r,\theta)\hat{n}^{a}_{\theta},\ \hat{\Phi}^{a}=\Phi^{a}/|\Phi|=h_1(r,\theta)\hat{n}^{a}_{r}+h_2(r,\theta)\hat{n}^{a}_{\theta}. \qquad (2)$$

Here the isospin unit vectors are given by

$$\hat{n}^{a}_{r}=\sin\theta\,\cos n\phi\,\delta^{a}_{1}+\sin\theta\,\sin n\phi\,\delta^{a}_{2}+\cos\theta\,\delta^{a}_{3},$$

$$\hat{n}^{a}_{\theta}=\cos\theta\,\cos n\phi\,\delta^{a}_{1}+\cos\theta\,\sin n\phi\,\delta^{a}_{2}-\sin\theta\,\delta^{a}_{3},$$

$$\hat{n}^{a}_{\phi}=-\sin n\phi\,\delta^{a}_{1}+\cos n\phi\,\delta^{a}_{2}. \qquad (3)$$

The definition of spatial unit vectors $\hat{r}_i,\hat{\theta}_i$ and $\hat{\phi}_i$ is similar to Eq. (4) when $n=1$. Now, using the definitions of $\cos\alpha=h_1(r,\theta)\cos\theta-h_2(r,\theta)\sin\theta$ and $\cos\kappa=\frac{\sin\theta}{n}\left(h_2(r,\theta)\psi_2-h_1(r,\theta)R_2\right)$ and $\gamma=\cos\alpha+\cos\kappa$, the 't Hooft's magnetic field becomes $B_i=-n\epsilon_{ijk}\partial_j\gamma\,\partial_k\phi$, where the lines of $\gamma=$ constant, on the vertical plane of $\phi=0$, will represent the magnetic field lines. Also, the unit vectors of magnetic field are given by $\hat{B}_i = \dfrac{r\partial_r(\gamma)\hat{\theta}_i - \partial_\theta(\gamma)\hat{r}_i}{\sqrt{(r\partial_r(\gamma))^2 + (\partial_\theta(\gamma))^2}}$. Since the gauge field is time independent, the Abelian electric field is $E_i = \partial_i A_0 = \partial_i(\tau_1(r,\theta)h_1(r,\theta) + \tau_2(r,\theta)h_2(r,\theta))$ and therefore, the unit vectors of magnetic field become $\hat{E}_i = \dfrac{r\partial_r A_0 \hat{r}_i + \partial_\theta A_0 \hat{\theta}_i}{\sqrt{(r\partial_r A_0)^2 + (\partial_\theta A_0)^2}}$. Now, the total electric charge of the system, Q can be evaluated numerically by

$$Q = \frac{1}{4\pi\xi}\int \partial^i E_i \, d^3x. \tag{4}$$

3. The numerical solutions and results

Equations of motion (\ref{eq.1}) with the gauge field and Higgs field of *Ansatz* (2), lead us to a system of eight coupled nonlinear second order equations. For a three-pole MAC solution the boundary conditions at large distances are given by [1]:

$$\psi_1(r,\theta)|_{r\to\infty} = 3,\ \psi_2(r,\theta)|_{r\to\infty} = \frac{n(\sin\theta + \cos\theta\, \sin 2\theta)}{\sin\theta} = n(\cos 2\theta + 2),$$

$$R_1(r,\theta)|_{r\to\infty} = 0,\ R_2(r,\theta)|_{r\to\infty} = \frac{n(\cos\theta - \cos\theta\, \cos 2\theta)}{\sin\theta} = n\sin 2\theta,$$

$$\Phi_1(r,\theta)|_{r\to\infty} = \xi\cos 2\theta, \quad \Phi_2(r,\theta)|_{r\to\infty} = \xi\sin 2\theta,$$

$$\tau_1(r,\theta)|_{r\to\infty} = \eta\xi\cos 2\theta, \quad \tau_2(r,\theta)|_{r\to\infty} = \eta\xi\sin 2\theta\,.$$

The Higgs field and the time component of the gauge are supposed to be parallel in isospin space at large distances. The trivial boundary conditions at $r = 0$ are given by [1], [3]:

$$\psi_1 = \psi_2 = R_1 = R_2 = 0,$$

$$\sin\theta\,\tau_1(0,\theta) + \cos\theta\,\tau_2(0,\theta) = 0,\ \sin\theta\,\Phi_1(0,\theta) + \cos\theta\,\Phi_2(0,\theta) = 0,$$

$$\partial_r(\cos\theta\,\tau_1(r,\theta) - \sin\theta\,\tau_2(r,\theta))|_{r=0} = 0,$$

$$\partial_r(\cos\theta\,\Phi_1(r,\theta) - \sin\theta\,\Phi_2(r,\theta))|_{r=0} = 0.$$

Along the z-axis we have [1], [3]:

$$R_A(r,\theta)|_{\theta\to 0,\pi} = \Phi_2(r,\theta)|_{\theta\to 0,\pi} = \tau_2(r,\theta)|_{\theta\to 0,\pi} = 0,$$

$$\partial_\theta \psi_A(r,\theta)|_{\theta\to 0,\pi} = \partial_\theta \Phi_1(r,\theta)|_{\theta\to 0,\pi} = \partial_\theta \tau_1(r,\theta)|_{\theta\to 0,\pi} = 0.$$

The numerical solution is based on trust region reflective algorithm and is performed by the fsolve package of MATLAB.

As is illustrated in Figure (1) along the higher energy bifurcating branch two transitions occur. For $\eta = 0.5$, the second transition occurs at $\lambda = 20.83$. At this critical point, the configuration changes from three poles and two rings to the new configuration of one pole (at origin) and two rings.

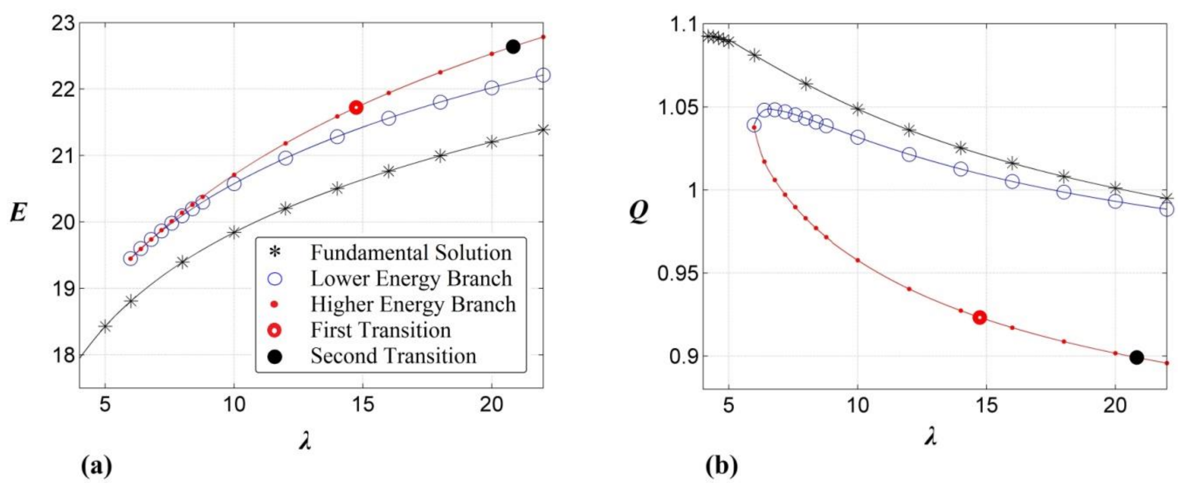

Figure 1. Plots of (a) the total energy, E, and (b) the total electric charge, Q, versus the Higgs self-coupling, λ, when $n=4$, $\eta=0.5$. For this case, the bifurcation occurs at $\lambda=5.979$. Also two transition points are shown along the higher energy branch at $\lambda=14.74$ and $\lambda=20.83$.

Figure (2) shows the evolution of electric and magnetic fields during these transitions. The direction of the unit vectors shows that the magnetic charge of the pole which is located at the origin changes at the second transition point and the negative magnetic charge of this pole during a charge transition becomes positive. Also a similar transition occurs for electric charge of this pole. The electric charge of the dyon which is located at the origin is negative for smaller values of λ. That is while after the second transition for $\lambda > 20.83$ (and $\eta = 0.5$) the electric charge of this dyon becomes positive as well.

Also, integration on very small volume including the origin for higher energy branch using the Eq. (4) shows that the electric charge of the dyon which is located at the origin, vanishes at the point of second transition.

4. Conclusion

This study indicates that some geometrical transitions would cause a major transition in magnetic and electric charge of the dyons in MAC configuration to happen. During these charge transitions, negative magnetic and electric charges of the dyon which is located at the origin change to positive magnetic and electric charges.

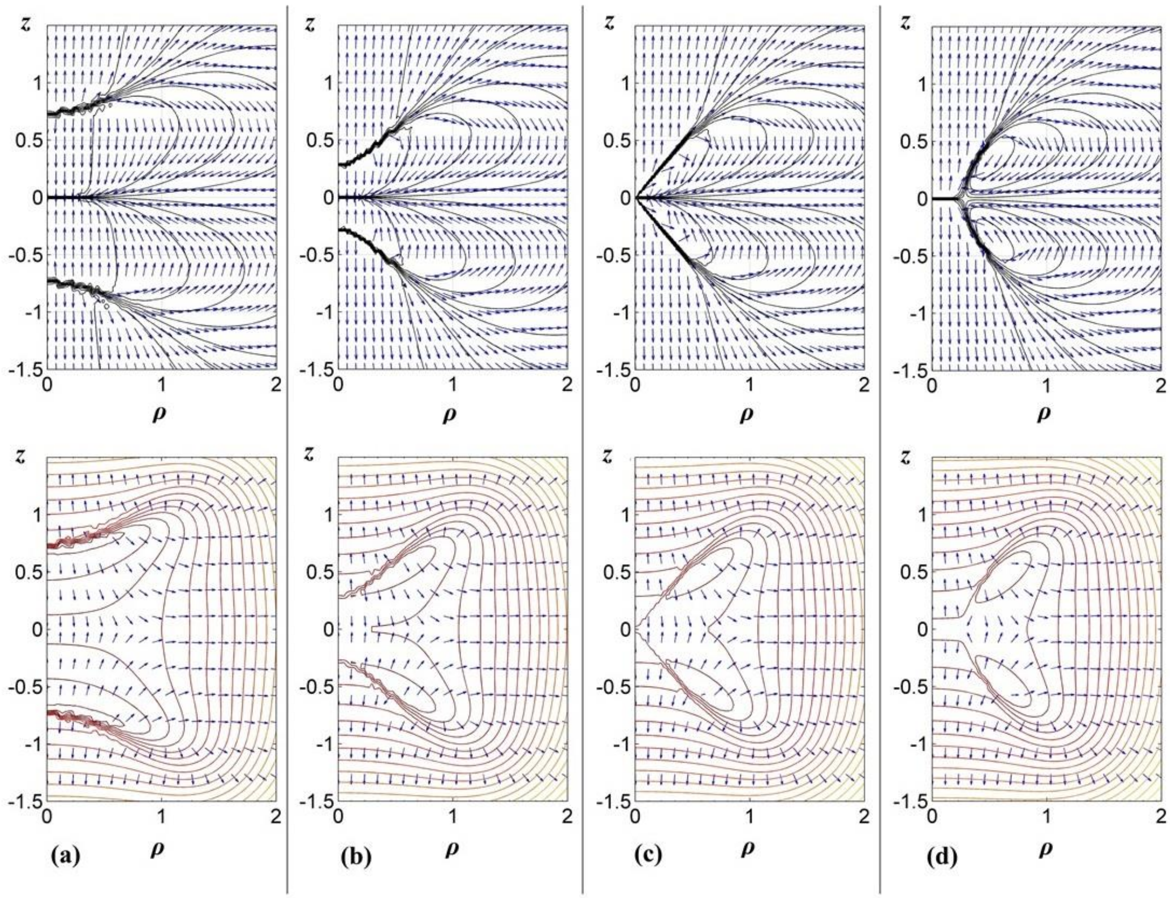

Figure2. Magnetic field lines and magnetic field's unit vectors (top) and equipotential lines and unit vectors of electric field (bottom) of the HEB solution for the case of $n=4$, $\eta=0.5$. The cases of (a) $\lambda=10$, with three poles, (b) $\lambda=18$, with two rings and three poles (after the first transition), (c) $\lambda=20.83$, where the second transition occurs and (d) $\lambda=30$, with one pole and two rings, are shown.

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